MODELING THE FLOW OF RESOURCES IN A BALANCED ECONOMIC SYSTEM: FROM A KEYNESIAN PERSPECTIVE

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Abstract

This paper explores the analogy between the dynamic flow of fluids and the movement of resources in an economy. Using concepts inspired by mathematical models of generalized Oldroyd-B fluids, the study attempts to develop a conceptual framework that connects the stability of fluid motion with economic equilibrium. The paper extends this perspective through Keynesian theory, focusing on income, consumption, savings, and investment as interrelated flows that determine macroeconomic balance. The goal is to show that both systems: physical and economic, seek stability through internal adjustments and external pressures. The interdisciplinary comparison contributes to a better understanding of economic equilibrium and highlights the value of mathematical thinking in the interpretation of economic processes.

Keywords: Oldroyd-B fluid, parallel plates, the Keynesian framework, economic system.

JEL Classification: E12, O41, C02.

1. Introduction

The study of equilibrium is central to both physics and economics. In physics, fluid models describe how forces and pressures interact to create balance and motion. In economics, equilibrium represents the stable relationship between production, income, and expenditure. Although these fields appear distant, their underlying mechanisms share a common principle: the continuous adjustment of flows.

The Oldroyd-B model, used in fluid mechanics, examines how materials with both viscous and elastic properties react to pressure and deformation. Similarly, in economics, the flow of income and capital reacts to fiscal and monetary pressures. When the system faces disturbances, such as inflation, unemployment, or external shocks, it tends to establish equilibrium through adjustments in spending, savings, and investment.

This paper proposes a conceptual bridge between these two domains. By interpreting the flow of economic variables through mathematical logic inspired by fluid dynamics, we can better visualize how stability is formed and maintained in an economy. The Keynesian

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model provides the theoretical foundation for this analogy, as it describes how total output, consumption, and investment interact dynamically to determine national income.

2. Mathematical and Conceptual Background

In the Oldroyd-B model, fluid flow depends on several parameters that represent viscosity, elasticity, and external forces. The system is considered stable when small changes in pressure or boundary conditions do not cause large variations in flow. In simple terms, the model explains how a fluid responds to stress while maintaining an overall balance.

If we translate this concept to economics, we can interpret "viscosity" as the resistance of the economy to change factors such as institutional rigidity, regulation, or labor market constraints. "Elasticity" reflects the adaptability of markets and agents: how quickly consumption or investment responds to shifts in income or expectations.

A balanced flow in fluid mechanics corresponds to a stable macroeconomic environment. Just as a fluid seeks a steady state movement between two parallel plates, an economy seeks a level of output and income where total demand equals total supply. Both systems are defined by feedback processes that drive them toward equilibrium.

The flows between parallel plates for different non-newtonian fluids are studied by many authors. In [11]-[20] studied the unsteady unidirectional flow of an Oldroyd-B fluid.

Hayat et al. [1] obtained exact analytical solutions for unsteady flow between parallel plates of an Oldroyd-B fluid. They showed the existence of solution for each of the six problems considered :1) unsteady Couette flow, 2) unsteady flow for a rigid and free boundaries, 3) flow between parallel plates suddenly set in motion with same speed, 4) unsteady Poiseuille flow, 5) unsteady generalized Couette flow and 6) unsteady generalized Couette flow for rigid and free boundaries. They showed the existence of solution for each problem considered for all values of non-Newtonian parameters. They also represented velocity profile for small and large times.

Two types of unsteady unidirectional flows of a generalized Oldroyd-B fluid with fractional derivative are presented in [11]. The velocity distrubutions are determinated by discrete Laplace transform and finite Fouriei sine transform.

In [12]-[13] Is studied the steady unidirectional flow of an Oldroyd 6-constant fluid. In [12] is studied Oldroyd 6-constant fluid, flowing through a channel under the as sumption that the fluid slips on the wall. The analitic solutions for Couette, Poiseuille and generalized Couette flows are found using homotopy analysis method (HAM).

The same method is used in [13], to study the steady flow of Oldroyd 6-constant fluid. To find analytical solutions is used a technique based on homotopy (HAM), is successfully applied to several non-linear problems.

Flow between two parallel plates for a special model of a generalized fluid of complexity two, which includes the Navier-Stokes fluid, power-law fluid, classical second grade-fluid is analyzed in [14]. Here the assumption is that the flow meets Navier slip conditions at the boundary.

For second grade fluids are studied various types of unsteady flows between parallel plates. In [15] are considered the unidirectional flows of a fluid of second grade. These flows are produced by sudden application of a constant pressure gradient or by the impulsive motion of one or two boundaries. The other property of unsteady flows described in

[15] is that the no-slip boundary condition is sufficient for unsteady flows, but for steady flows an additional condition is required.

In [16] are investigated two types of unsteady flow of a second-grade fluid: flow in a duct and flow in a rectangular cross-section. The effects of the side walls on the unsteady flow of such fluid in a duct of rectangular cross-section are considered.

Asghar et al. in [17] considered two problems of unidirectional flow involving second grade fluid with variable material properties. They showed that some exact analytical solutions are possible for unsteady Couette flows of second grade fluid in terms of trigonometric and Chebyshev functions.

In [18] is studied the unsteady motion of inhomogeneous incompressible fluid, where the viscosity varies spatially. Two types of flow are analyzed in this paper, namely: Stokes' second problem and the flow between two parallel plates where one is oscillating.

In this paper the problem of the flow of a generalized Oldroyd -B fluid is studied between two parallel plates. We are interested in knowing if von Karman type solutions are admissible for this fluid. We solve the problem using a power series technique and obtain the velocity field as well as the components for the Cauchy tensor.

1.The flow problem.

The Cauchy stress tensor is given:

$$\mathrm{T} = -p\mathrm{I} + \mathrm{T}_E, \quad \mathrm{T}_E + \lambda \frac{D\mathrm{T}_E}{Dt} = \mu\left(\mathrm{A}_1 + \lambda_1 \frac{D\mathrm{A}_1}{Dt}\right), \tag{1.1}$$

where T_E is the extra-stress tensor (effective stress - tensor), -pI denotes the inde- terminate spherical stress, $\mathrm{A}_1 = \mathrm{L} + \mathrm{L}^T$ is the first Rivlin - Ericksen tensor, L is the velocity gradient, λ and λ_1 are the relaxation times, μ is the dynamic viscosity. The convective derivative of T_E and A_1 are expressed through:

$$\begin{aligned} \frac{D\mathbf{T}_E}{Dt} &= \dot{\mathbf{T}}_E + \mathbf{T}_E\mathbf{L} + \mathbf{L}^T\mathbf{T}_E, \\ \frac{D\mathbf{A}_1}{Dt} &= \dot{\mathbf{A}}_1 + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1. \end{aligned} \tag{1.2}$$

The fluid flows between two parallel plates. The upper plate is supposed to be porous, and the fluid passes through with constant vertical velocity, meaning:

$\mathrm{v}\,(x, y)\,|_{y=d} = -v_0\mathrm{e}_2$

The other one moves with speed:

$\mathrm{v}\,(x, y)\,|_{y=0} = cx\mathrm{e}_1$

where" d" is the distance between the two plates, e_1 and e_2 are the unit vector in the horizontal and respective vertical directions and" c" is a given constant.

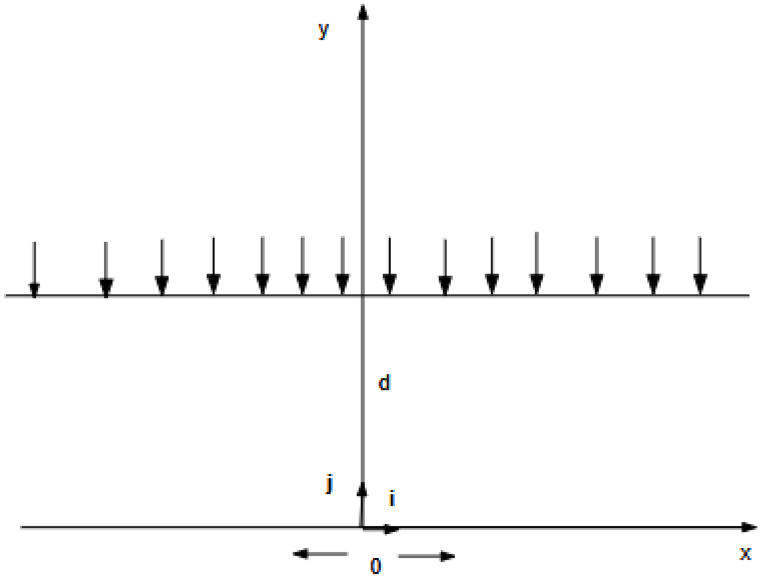

Figure 1: Geometric representation of the flow domain[4].

We are interested in the following problem: does a generalized Oldroyd -B fluid accept a von Karman solution for the flow problem described above? We shall suppose, like in [11], that the admissible velocity field is of von Karman type

$$u = cxf'(\eta),\ v = -cdf(\eta), \qquad (1.3)$$

where $y \equiv d\eta$.
This flow field satisfies the constraint of incompressibility: $div\, \mathrm{v} = 0$.
From the form of velocity field proposed results the acceleration

$$\mathbf{a} = dc^2\overline{x}(f'^2 - ff'')\mathrm{e}_1 + dc^2 ff'\mathrm{e}_2, \qquad (1.4)$$

where $\overline{x} = x/d$. The local form of the balance of linear momentum:

$$\rho \mathrm{a} = \rho \mathrm{b} + div\mathrm{T}.$$

We assume that the specific body force $\mathrm{b} \equiv 0$. Since the velocity field is independent of z, the stress field will also be independent of z. We can now write the equations of motion in the form

[4] Author's own contribution

$$\rho c^2\bar{x}d(f'^2 - ff'')\mathrm{e}_1 + \rho c^2 d f f' \mathrm{e}_2 = \frac{1}{d}(\frac{\partial T^{11}}{\partial \bar{x}} + \frac{\partial T^{12}}{\partial \eta})\mathrm{e}_1 + \frac{1}{d}(\frac{\partial T^{12}}{\partial \bar{x}} + \frac{\partial T^{22}}{\partial \eta})\mathrm{e}_2 +$$
$$+\frac{1}{d}(\frac{\partial T^{13}}{\partial \bar{x}} + \frac{\partial T^{23}}{\partial \eta})\mathrm{e}_3. \qquad (1.5)$$

The dimensionless form of the equation (1.5)

$$Re\bar{x}(f'^2 - ff'')\mathrm{e}_1 + Reff'\mathrm{e}_2 = (\frac{\partial \bar{T}^{11}}{\partial \bar{x}} + \frac{\partial \bar{T}^{12}}{\partial \eta})\mathrm{e}_1 + (\frac{\partial \bar{T}^{12}}{\partial \bar{x}} + \frac{\partial \bar{T}^{22}}{\partial \eta})\mathrm{e}_2 +$$
$$+(\frac{\partial \bar{T}^{13}}{\partial \bar{x}} + \frac{\partial \bar{T}^{23}}{\partial \eta})\mathrm{e}_3.$$

where $Re = \frac{\rho c d^2}{\mu}$ is the Reynolds number and $\bar{T}^{ij} = \frac{T^{ij}}{\mu c}$ are the dimensionless components of the Cauchy stress tensor.

Considering the expression $(1.1)_1$, we obtain the following system for the equations of motion

$$Re\bar{x}(f'^2 - ff'') = \frac{\partial \bar{T}_E^{11}}{\partial \bar{x}} + \frac{\partial \bar{T}_E^{12}}{\partial \eta} - \frac{\partial \bar{p}}{\partial \bar{x}},$$
$$Reff' = \frac{\partial \bar{T}_E^{12}}{\partial \bar{x}} + \frac{\partial \bar{T}_E^{22}}{\partial \eta} - \frac{\partial p}{\partial \eta}, \qquad (1.6)$$
$$\frac{\partial \bar{T}_E^{13}}{\partial \bar{x}} + \frac{\partial \bar{T}_E^{23}}{\partial \eta} = 0.$$

where $\bar{p} = \frac{p}{\mu c}$.

The constitutive equation (1.1) using (1.2) is equivalent to the following system

$$2f' - 2c\lambda_1(ff'' - 2f'^2) = \lambda c\left[\frac{x}{d}f'\frac{\partial \bar{T_E}^{11}}{\partial \bar{x}} - f\frac{\partial \bar{T_E}^{11}}{\partial \eta} + 2f'\bar{T_E}^{11}\right] + \bar{T_E}^{11},$$

$$\frac{x}{d}f'' + c\lambda_1\frac{x}{d}(3f'f'' - ff''') = \lambda c\left[\frac{x}{d}f'\frac{\partial \bar{T_E}^{12}}{\partial \bar{x}} - f\frac{\partial \bar{T_E}^{12}}{\partial \eta} + \frac{x}{d}f''\bar{T_E}^{11}\right] + \bar{T_E}^{11},$$

$$\lambda c\left[\frac{x}{d}f'\frac{\partial \bar{T_E}^{13}}{\partial \bar{x}} - f\frac{\partial \bar{T_E}^{13}}{\partial \eta} + f'\bar{T_E}^{13}\right] + \bar{T_E}^{13} = 0,$$

$$-2f' + 2c\lambda_1(ff'' + 2f'^2) = \lambda c\left[\frac{x}{d}f'\frac{\partial \bar{T_E}^{22}}{\partial \bar{x}} - f\frac{\partial \bar{T_E}^{22}}{\partial \eta} + 2\frac{x}{d}f''\bar{T_E}^{12} - 2f'\bar{T_E}^{22}\right] + \bar{T_E}^{22},$$

$$\lambda c\left[\frac{x}{d}f'\frac{\partial \bar{T_E}^{23}}{\partial \bar{x}} - f\frac{\partial \bar{T_E}^{23}}{\partial \eta} + \frac{x}{d}f''\bar{T_E}^{13} - f'\bar{T_E}^{23}\right] + \bar{T}_E^{23} = 0,$$

$$\lambda c\left[\frac{x}{d}f'\frac{\partial \bar{T_E}^{33}}{\partial \bar{x}} - f\frac{\partial \bar{T_E}^{33}}{\partial \eta}\right] + \bar{T_E}^{33} = 0. \qquad (1.7)$$

The dimensionless boundary conditions obtained from the mechanical problem described

$$f(0) = 0, \quad f(1) = \frac{v_0}{cd}, \quad f'(0) = 1, \quad f'(1) = 0. \qquad (1.8)$$

2. The Solution for the equations of motion.

We are looking for a solution to the problem (1.6) -(1.7) -(1.8).
For the components of the effective stress, we make the folowing assumptions

$$\bar{T}_E^{13} = \bar{T}_0^{13}(\eta) + \frac{x}{d}\bar{T}_1^{13}(\eta), \quad \bar{T}_E^{23} = \bar{T}_0^{11}(\eta) + \frac{x}{d}\bar{T}_1^{23}(\eta),$$
$$\bar{T}_E^{33} = \bar{T}_0^{11}(\eta) + \frac{x}{d}\bar{T}_1^{33}(\eta), \quad \bar{T}_E^{11} = \bar{T}_0^{11}(\eta) + \frac{x}{d}\bar{T}_1^{11}(\eta), \qquad (2.1)$$
$$\bar{T}_E^{12} = \bar{T}_0^{12}(\eta) + \frac{x}{d}\bar{T}_1^{12}(\eta), \quad \bar{T}_E^{22} = \bar{T}_0^{22}(\eta) + \frac{x^2}{d^2}\bar{T}_2^{22}(\eta).$$

First, we analyze the system consisting of equations $(1.7)_{3,5,6}$ and $(1.6)_3$.

If we use the assumptions $(2.1)_{1,2,3}$ and identify the coefficients of $\left(\frac{x}{d}\right)^n (n = 0,1,2)$, we obtain the following system

$$\lambda c\left[f'\bar{T}_1^{13} - f\frac{d\bar{T}_1^{13}}{d\eta} + f'\bar{T}_1^{13}\right] + \bar{T}_1^{13} = 0, \quad \lambda c\left[-f\frac{d\bar{T}_0^{13}}{d\eta} + f'\bar{T}_0^{13}\right] + \bar{T}_0^{13} = 0,$$

$$c\lambda f''\bar{T}_1^{13} = 0, \quad \lambda c\left[-f\frac{d\bar{T}_1^{23}}{d\eta} + f''\bar{T}_0^{13}\right] + \bar{T}_1^{23} = 0,$$

$$-\lambda c\left[f\frac{d\bar{T}_0^{23}}{d\eta} + f'\bar{T}_0^{23}\right] + \bar{T}_0^{23} = 0, \quad \lambda c\left[f'\bar{T}_1^{33} - f\frac{d\bar{T}_1^{33}}{d\eta}\right] + \bar{T}_1^{33} = 0,$$

$$-\lambda c f\frac{d\bar{T}_0^{33}}{d\eta} + \bar{T}_0^{33} = 0, \quad \frac{d\bar{T}_1^{23}}{d\eta} = 0, \quad \frac{d\bar{T}_0^{23}}{d\eta} + \bar{T}_1^{13} = 0. \tag{2.2}$$

From $(2.2)_3$result $\bar{T}_1^{13} = 0$, ,because the equation $f'' = 0$ has no solutions if we consider the bilocal problem obtained from the described mechanical problem: $f(0) = 0, f(1) = \frac{v0}{cd}, f'(0) = 1, f'(1) = 0.$ Also obtain that $\bar{T}^{23} = 0.$

The components of the effective stress are now expressed as

$\bar{T}^{13} = \bar{T}^{13}(\eta),\ \bar{T}^{23} = {}^{x}\bar{T}^{23}(\eta),\ \bar{T}^{33} = T^{33}(\eta) + {}^{x}\bar{T}^{33}(\eta).$

$$\bar{T}_E^{13} = \bar{T}_0^{13}(\eta), \quad \bar{T}_E^{23} = \frac{x}{d}\bar{T}_1^{23}(\eta), \quad \bar{T}_E^{33} = T_0^{33}(\eta) + \frac{x}{d}\bar{T}_1^{33}(\eta). \tag{2.3}$$

The system (2.2) is now

$$\lambda c\left[-f\frac{d\bar{T}_0^{13}}{d\eta} + f'\bar{T}_0^{13}\right] + \bar{T}_0^{13} = 0, \quad \lambda c f''\bar{T}_0^{13} + \bar{T}_1^{23} = 0,$$

$$\lambda c\left[f'\bar{T}_1^{33} - f\frac{d\bar{T}_1^{33}}{d\eta}\right] + \bar{T}_1^{33} = 0, \quad -\lambda c f\frac{d\bar{T}_0^{33}}{d\eta} + \bar{T}_0^{33} = 0, \quad \frac{d\bar{T}_1^{23}}{d\eta} = 0. \tag{2.4}$$

Without loss of generality, we can suppose that $\bar{T}^{13} = \bar{T}^{23} = \bar{T}^{33} = 0$ for $\eta = 1$. Using these assumptions, we obtain that

$$\bar{T}_E^{13}(\bar{x},\eta) = \bar{T}_E^{23}(\bar{x},\eta) = \bar{T}_E^{33}(\bar{x},\eta) = 0.$$

Now we use the expressions $(2.1)_{4,5,6}$ in $(1.7)_{1,2,4}$ and we identify the coefficients of $\left(\frac{x}{d}\right)^n$, (n=0,1,2) Thus we obtain the following system for the constitutive equations

$$\lambda\left[3cf'\bar{T}_1^{11}-cf\frac{d\bar{T}_1^{11}}{d\eta}\right]+\bar{T}_1^{11}=0,$$

$$2c\mu f'-2c^2\mu\lambda_1(ff''-2f'^2)=\lambda\left[-cf\frac{d\bar{T}_0^{11}}{d\eta}+2cf'\bar{T}_0^{11}\right]+\bar{T}_0^{11},\quad c\lambda f''\bar{T}_1^{11}=0,$$

$$c\mu f''+c^2\mu\lambda_1(3f'f''-ff''')=\lambda\left[cf'\bar{T}_1^{12}-cf\frac{d\bar{T}_1^{12}}{d\eta}+cf''\bar{T}_0^{11}\right]+\bar{T}_1^{12},$$

$$-\lambda cf\frac{d\bar{T}_0^{12}}{d\eta}+\bar{T}_0^{12}=0,\quad 2c^2\mu\lambda_1f''^2=\lambda\left[-cf\frac{d\bar{T}_2^{22}}{d\eta}+2cf''\bar{T}_1^{12}\right]+\bar{T}_2^{22},$$

$$2c\lambda f''T_0^{12}=0,\quad -2c\mu f'+2c^2\mu\lambda_1(ff''+2f'^2)=\lambda\left[-cf\frac{d\bar{T}_0^{22}}{d\eta}-2cf'\bar{T}_0^{22}\right]+\bar{T}_0^{22}.$$

(2.5)

From $(2.5)_3$ and $(2.5)_7$ result $\bar{T}^{11}=0$ and $\bar{T}^{12}=0$. We notice that equations $(2.5)_1$ and $(2.5)_5$ are satisfayed. The components of the effective stress are now expressed as

$$\bar{T}_E^{11}=\bar{T}_0^{11}(\eta),\quad \bar{T}_E^{12}=\frac{x}{d}\bar{T}_1^{12}(\eta),\quad \bar{T}_E^{22}=\bar{T}_0^{22}(\eta)+\frac{x^2}{d^2}\bar{T}_2^{22}(\eta). \tag{2.6}$$

The system (2.5) becomes now

$$2f' - 2c\lambda_1(ff'' - 2f'^2) = \lambda c\left[-f\frac{d\bar{T_0}^{11}}{d\eta} + 2f'\bar{T_0}^{11}\right] + \bar{T_0}^{11},$$

$$f'' + c\lambda_1(3f'f'' - ff''') = \lambda c\left[f'\bar{T_1}^{12} - f\frac{d\bar{T_1}^{12}}{d\eta} + f''\bar{T_0}^{11}\right] + \bar{T_1}^{12},$$

$$2c\lambda_1 f''^2 = \lambda c\left[-f\frac{d\bar{T_2}^{22}}{d\eta} + 2f''\bar{T_1}^{12}\right] + \bar{T_2}^{22}, \qquad (2.7)$$

$$-2f' + 2c\lambda_1(ff'' + 2f'^2) = \lambda c\left[-f\frac{d\bar{T_0}^{22}}{d\eta} - 2f'\bar{T_0}^{22}\right] + \bar{T_0}^{22}.$$

We eliminate the pressure in the system (1.6), and we use (2.6), thus obtaining the following equation

$$Re(f'f'' - ff''') = \frac{d^2\bar{T}_1^{12}}{d\eta^2} - 2\frac{d\bar{T}_2^{22}}{d\eta}. \qquad (2.8)$$

The problem that we must solve now is (2.7) -(2.8). The unknown functions are: $\mathbf{f}, \bar{T_1}^{12}, \bar{T_2}^{22}, \bar{T_0}^{22}, \bar{T_0}^{11}$.
The boundary conditions for fa re (1.8) and for the components of the effective stress is

$$\bar{T_1}^{12}(1) = 2(1 - 3\frac{v_0}{cd}),$$

$$\bar{T_0}^{22}(1) = \bar{T_0}^{11}(1) = \bar{T_2}^{22}(1) = 0. \qquad (2.9)$$

We seek the solutions of the system by a double scale asymptotic development. The two parameters that appear in this development are $\beta << 1$ and $re << 1$. We first develop asymptotic compared with β the unknown functions from the system (2.7) − (2.8)

$$f = \sum_{i=0}^{\infty}\beta^i f_i, \qquad \bar{T_k}^{ij} = \sum_{l=0}^{\infty}\beta^l \bar{T}_{k(1)}^{ij}, \qquad (2.10)$$

where
$\beta \equiv O(c\lambda) = O(c\lambda_1) << 1.$

The system for zero-order approximations in β is

$$2f_0' = \bar{T}^{11}_{0(0)},$$
$$f_0'' = \bar{T}^{12}_{1(0)},$$
$$-2c\mu f_0' = \bar{T}^{22}_{0(0)}, \tag{2.11}$$
$$\bar{T}^{22}_{2(0)} = 0,$$
$$Re(f_0' f_0'' - f_0 f_0''') = (\bar{T}^{12}_{1(0)})''.$$

The conditions for approximations $f_0, f_i (i = 1, n)$ are

$$f_0(0) = 0, \quad f_0(1) = \frac{v_0}{cd}, \quad f_0'(0) = 1, \quad f_0'(1) = 0.$$
$$f_i(0) = f_i(1) = f_i'(0) = f_i'(1) = 0. \tag{2.12}$$

We derive twice in $(2.11)_2$ and introduced in $(2.11)_5$ we get the differential equation of order iv for f0

$$f_0^{iv} = Re(f_0' f_0'' - f_0 f_0'''). \tag{2.13}$$

If we make the following developments

$$f_0 = f_{00} + Re f_{01} + \ldots,$$
$$\bar{T}^{ij}_{k(l)} = \bar{T}^{ij}_{k(l0)} + Re\bar{T}^{ij}_{k(l1)} + \ldots. \tag{2.14}$$

Thus, we obtain a seguence of linear problems whose solutions are polynomials in $\eta \in [0, 1]$.

$$f_{00}^{iv} = 0,$$

$$f_{01}^{iv} = f'_{00} f''_{00} - f_{00} f'''_{00},$$

$$f_{02}^{iv} = f'_{00} f''_{01} + f'_{01} f''_{00} - f_{00} f'''_{01} - f_{01} f'''_{00}, \tag{2.15}$$

...

Thus, for the first two approximations of f_0 (in Re) we obtain

$$f_{00}(\eta) = A\eta^3 + B\eta^2 + \eta,$$

$$f_{01}(\eta) = \frac{A^2}{70}\eta^7 + \frac{AB}{30}\eta^6 + \frac{B^2}{30}\eta^5 + \frac{B}{12}\eta^4 +$$

$$+\left(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6}\right)\eta^3 + \left(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12}\right)\eta^2. \tag{2.16}$$

With

$$A = 1 - 2\frac{v_0}{cd}, \quad B = 3\frac{v_0}{cd} - 2, \tag{2.17}$$

For the approximations we obtain (using $(2.16)_1$) the following formula

$$\bar{T}^{11}_{0(00)} = 2(3A\eta^2 + 2B\eta + 1),$$

$$\bar{T}^{12}_{1(00)} = 6A\eta + 2B,$$

$$\bar{T}^{22}_{0(00)} = -2(3A\eta^2 + 2B\eta + 1), \tag{2.18}$$

and for $\bar{T}^{ij}_{k(l1)}$ we have (using $(2.16)_2$)

$$\bar{T}^{11}_{0(01)} = 2\left\{\frac{A^2}{10}\eta^6 + \frac{AB}{5}\eta^5 + \frac{B^2}{6}\eta^4 + \frac{B}{3}\eta^3 + 3(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6})\eta^2 + 2(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12})\eta\right\}.$$

$$\bar{T}^{12}_{1(01)} = \left\{\frac{3A^2}{5}\eta^5 + AB\eta^4 + \frac{2B^2}{3}\eta^3 + B\eta^2 + 6(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6})\eta + 2(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12})\right\},$$

$$\bar{T}^{22}_{0(01)} = -2\left\{\frac{A^2}{10}\eta^6 + \frac{AB}{5}\eta^5 + \frac{B^2}{6}\eta^4 + \frac{B}{3}\eta^3 + 3(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6})\eta^2 + 2(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12})\eta\right\},$$

$$\bar{T}^{22}_{2(11)} = 0. \qquad (2.19)$$

For the first-order approximations in beta: f_1 and $\bar{T}^{ij}$ we can show that

$$f_1 \cong f_{10} + Re f_{11} = 0,$$

$$\bar{T}^{ij}_{k(1)} \cong \bar{T}^{ij}_{k(10)} + Re\bar{T}^{ij}_{k(11)} = 0.$$

Consequently, from double scale asymptotic development of the unknown function f

$$f = \sum_{i,j=0}^{n}(f_{ij}\beta^i)Re^j = \sum_{j=0}^{n} f_{0j}Re^j + \beta\sum_{j=0}^{n} f_{1j}Re^j + \ldots = (f_{00}+\beta f_{10}) + (f_{01}+\beta f_{11})Re + \ldots,$$

we make the following approximation

$$f \cong f_{00} + Re f_{01}. \qquad (2.20)$$

Similarly for the dimensionless components of the effective stress we consider

$$\bar{T}_0^{11} \cong \bar{T}_{0(00)}^{11} + Re\bar{T}_{0(01)}^{11}$$

$$\bar{T}_1^{12} \cong \bar{T}_{1(00)}^{12} + Re\bar{T}_{1(01)}^{12},$$

$$\bar{T}_2^{22} \cong 0, \qquad (2.21)$$

$$\bar{T}_0^{22} \cong \bar{T}_{0(00)}^{22} + Re\bar{T}_{0(01)}^{22}.$$

Conclusions. We evaluate the interaction between the fluid and the two plates, that is $\mathrm{Tn} \cdot \mathrm{n}$ and $\mathrm{Tn} \cdot \tau$. If we use $(2.6)_3$

$$T_E^{22} = T_0^{22}(\eta) + \frac{x^2}{d^2} T_2^{22}(\eta),$$

and if we are considering that we find that $\bar{T}_2^{22} \cong 0$ find that

$$\mathbf{Tn} \cdot \mathbf{n} = -\bar{p} + \bar{T}_0^{22}(\eta).$$

Using $(2.6)_3$, $(2.18)_3$ and $(2.19)_3$ we obtain

$$\mathbf{Tn} \cdot \mathbf{n} = -p + \bar{T}_{0(00)}^{22} + Re\bar{T}_{0(01)}^{22} = -\bar{p} - 2\left\{3A\eta^2 + 2B\eta + 1\right.$$
$$+Re\left[\frac{A^2}{10}\eta^6 + \frac{AB}{5}\eta^5 + \frac{B^2}{6}\eta^4 + \frac{B}{3}\eta^3 + 3(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6})\eta^2\right. \qquad (2.22)$$
$$\left.\left. + 2(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12})\eta\right]\right\}.$$

where A and B are given by (2.17). A simple calculus gives the force on the two plates. That is for normal forces

$$\mathbf{Tn} \cdot \mathbf{n} \mid_{\eta=0} = -\bar{p} - 2,$$
$$\mathbf{Tn} \cdot \mathbf{n} \mid_{\eta=1} = -\bar{p} - 2(3A + 2B + 1). \qquad (2.23)$$

It is easy to see that if $\frac{v0}{cd\,2} = \frac{1}{\_}$

$$(\mathbf{Tn} \cdot \mathbf{n} \,|_{\eta=0} + \bar{p}) = -2,$$

$$(\mathbf{Tn} \cdot \mathbf{n} \,|_{\eta=1} + \bar{p}) = 0.$$

Using (2.6) and (2.21) we have

$$\bar{T}^{11} + \bar{p} \equiv \bar{T_E}^{11} = \bar{T_0}^{11}(\eta) = \bar{T}^{11}_{0(00)} + Re\bar{T}^{11}_{0(01)},$$

$$\bar{T}^{12} + \bar{p} \equiv \bar{T_E}^{12} = \frac{x}{d}\bar{T_1}^{12}(\eta) = \frac{x}{d}(\bar{T}^{12}_{1(00)} + Re\bar{T}^{12}_{1(01)}).$$

Thus, we obtain the dimensionless components of the effective stress

$$\bar{T}^{11}_E = 2(3A\eta^2 + 2B\eta + 1) + 2Re\left\{\frac{A^2}{10}\eta^6 + \frac{AB}{5}\eta^5 + \frac{B^2}{6}\eta^4 + \frac{B}{3}\eta^3 + \right.$$
$$\left. + 3(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6})\eta^2 + 2(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12})\eta\right\},$$
$$\bar{T}^{12}_E = \frac{x}{d}\left\{6A\eta + 2B + Re\left[\frac{3A^2}{5}\eta^5 + AB\eta^4 + \frac{2B^2}{3}\eta^3 + B\eta^2 + \right.\right. \tag{2.24}$$
$$\left.\left. + 6(-\frac{A^2}{14} - \frac{2AB}{15} - \frac{B^2}{10} - \frac{B}{6})\eta + 2(\frac{2A^2}{35} + \frac{AB}{10} + \frac{B^2}{15} + \frac{B}{12})\right]\right\}.$$

3. The Keynesian Framework

Keynesian economics is built upon the idea that total income in an economy (Y) equals the sum of consumption (C), investment (I), government spending (G), and net exports (NX): Y = C + I + G + NX.

Each of these components represents a flow of resources. Income flows from firms to households; consumption flows from households back to firms; savings become the source of future investments; and government and trade activities act as external currents influencing the balance.

According to Keynes, equilibrium occurs when aggregate demand equals aggregate supply. However, the process is dynamic rather than static. If consumption increases faster than production, inflationary pressures emerge. If investment declines, unemployment may rise. The system continuously adjusts much like a fluid adjusting to new pressure gradients.

In this sense, we can view the Keynesian multiplier as an “elastic coefficient” that measures how changes in spending propagate through the economy. A high multiplier means that small changes in investment can produce large variations in total income, like how a low-viscosity fluid responds strongly to small forces.

The stability of equilibrium depends on the relationship between these flows. Excessive rigidity in consumption or savings behavior can slow recovery, while high adaptability enhances resilience. Therefore, the economic system: like a physical one, requires a balance between flexibility and structure.

Keynesian economics emphasizes that equilibrium in an economy is not a static condition but a continuous process of adjustment. When total spending (aggregate demand) equals total output (aggregate supply), the economy operates at its potential level. However, in real life, disturbances such as changes in consumer confidence, government policy, or global trade can shift these flows and move the system away from equilibrium.

Keynes identified consumption (C) and investment (I) as the two primary forces determining total income (Y). Consumption depends largely on disposable income and psychological factors influencing people's willingness to spend. Investment, on the other hand, depends on interest rates and the expected return on capital. When interest rates fall, investments tend to rise, stimulating production and income.

Savings (S) play a crucial role in this process. While savings provide the funds needed for investment, excessive savings can reduce demand, leading to stagnation, a concept Keynes referred to as the paradox of thrift. Maintaining equilibrium thus requires a balance between saving and spending, which can be influenced by fiscal policy (taxation and government expenditure) and monetary policy (control of money supply and interest rates).

This framework mirrors physical systems where the flow of forces and responses determines balance. Just as fluid pressure can be adjusted to restore stability, fiscal and monetary instruments can be used to correct economic imbalances. The objective of economic policy is therefore like the stabilization of a dynamic system: to reduce oscillations and guide the economy toward steady growth.

4. Integrating the Two Models

The analogy between fluid dynamics and economic equilibrium is not purely metaphorical. Both systems can be modeled through equations describing rates of change and feedback mechanisms. In the Oldroyd-B context, stress and strain rates define how energy is distributed within the system. In economics, similar interactions occur between production, income, and expenditure.

For example, when investment levels change, they create "pressure differentials" in income distribution and demand. These differentials trigger flows of adjustment: in wages, prices, and resource allocation, until a new equilibrium is achieved. A resistant economy, with high institutional viscosity, will adjust slowly, while a flexible one will stabilize faster.

By combining the structural insights of mathematics with the behavioral focus of Keynesian theory, this framework allows us to reinterpret economic equilibrium not as a fixed point but as a process of continuous motion. Stability emerges from constant adjustments, where opposing forces, spending and saving, investment and consumption; balance each other through dynamic flow.

This interdisciplinary approach highlights how economic systems can be studied using simplified versions of physical models, improving our understanding of how shocks propagate and how resilience can be enhanced through policy design.

5. Conclusions

The comparison between fluid flow and economic equilibrium reveals that both disciplines share a fundamental logic: systems strive for balance through the adjustment of internal flows. The Oldroyd-B model offers a mathematical metaphor for the behavior of macroeconomic variables, while Keynesian theory provides the economic meaning of those movements.

Viewing the economy as a dynamic system of interacting flows helps policymakers and researchers understand the importance of adaptability, coordination, and stability. Just as viscosity and elasticity determine whether fluid remains stable or turbulent, structural and behavioral factors determine whether an economy sustains growth or falls into crisis.

This interdisciplinary framework opens future directions for research, such as modeling capital mobility, digital resource flows, or policy transmission mechanisms, using mathematical reasoning inspired by the physical sciences.

This study proposed a conceptual analogy between the flow of resources in an economy and the flow of matter in a physical system. Through this comparison, it becomes clear that equilibrium: whether physical or economic, is not a fixed point but a state of dynamic balance maintained through continuous adjustments.

In the Keynesian framework, equilibrium is sustained by the interdependence of income, consumption, savings, and investment. When one of these components' changes, others must adapt to restore balance. Similarly, in fluid dynamics, any change in pressure or force generates compensatory movements that stabilize the system.

Understanding economic equilibrium as a process of motion rather than rest can help policymakers design more adaptive and flexible strategies. Instead of seeking perfect stability, economies should aim for resilient equilibrium, the ability to absorb shocks and return to sustainable growth without major distortions.

Moreover, the integration of mathematical reasoning, even at a conceptual level, offers a structured way to model how policies propagate through an economy. This interdisciplinary approach combining mathematical modeling and economic theory can support better decision-making in areas such as fiscal sustainability, financial stability, and digital transformation.

In conclusion, both economists and mathematicians share a common goal: to understand how systems respond to change. By studying these flows, we learn that equilibrium is not an absence of movement but the harmony of opposing forces.

References

[1] Keynes, J.M. — *The General Theory of Employment, Interest and Money.* Macmillan. 1936.

[2] Schwab, K. — *The Fourth Industrial Revolution.* World Economic Forum. 2016.

[3] Brynjolfsson, E.; McAfee, A. — *The Second Machine Age.* W.W. Norton & Company. 2014.

[4] Miao, F.; Holmes, W.; et al. — *Artificial Intelligence and Education: Guidance for Policymakers.* UNESCO Publishing. 2021.

[5] OECD — *The Digital Transformation of Education: Connecting Schools, Empowering Learners.* OECD Publishing. 2021.

[6] Tapscott, D. — *The Digital Economy: Rethinking Promise and Peril in the Age of Networked Intelligence.* McGraw-Hill. 2014.
[7] World Bank Group — *Digital Transformation in Romania: Country Diagnostic.* 2022.
[8] UNCTAD — *Technology and Innovation Report 2021: Catching Technological Waves – Innovation with Equity.* United Nations. 2021.
[9] Eurostat — *Digital Economy and Society Statistics – Households and Individuals.* Brussels. 2023.
[10] Hicks, J.R. - *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory.* Oxford University Press. 1946.
[11] Haitao, Q.; Minguyu Y.X. — *Some Unsteady Unidirectional Flows of a Generalized Oldroyd-B Fluid with Fractional Derivative. Applied Mathematical Modelling,* 33, pp. 4184–4191. 2009.
[12] Hayat, T.; Khan, M.; Ayub, M. — *On Non-Linear Flows with Boundary Condition. ZAMP,* 56, pp. 1012–1029. 2005.
[13] Hayat, T.; Khan, M.; Ayub, M. — *On the Explicit Analytic Solutions of an Oldroyd 6-Constant Fluid. International Journal of Engineering Science,* 42, pp. 123–135. 2004.
[14] Hron, J.; Le Roux, C.; Malek, J.; Rajagopal, K.R. — *Flows of Incompressible Fluids Subject to Navier's Slip on the Boundary. Computers and Mathematics with Applications,* 56, pp. 2128–2143. 2008.
[15] Erdogan, M.E.; Imrak, C.E. — *On Unsteady Unidirectional Flows of a Second-Grade Fluid. International Journal of Non-Linear Mechanics,* 40, pp. 1238–1251. 2005.
[16] Erdogan, M.E.; Imrak, C.E. — *Effects of the Side Walls on the Unsteady Flow of a Second-Grade Fluid in a Duct of Uniform Cross-Section. International Journal of Non-Linear Mechanics,* 39, pp. 1379–1384. 2004.
[17] Asghar, S.; Hayat, T.; Ariel, P.D. — *Unsteady Couette Flows in a Second-Grade Fluid with Variable Material Properties. Communications in Nonlinear Science and Numerical Simulation,* 14, pp. 154–159. 2009.
[18] Massoudi, M.; Vaidya, A. — *Unsteady Flows of Inhomogeneous Incompressible Fluids. International Journal of Non-Linear Mechanics,* doi:10.1016/j.ijnonlinmec.2011.02.006. 2011.
[19] Verma, V.D.; Singh, R.; Sharma, R. — *Ind. J. Tech,* 24, pp. 557–564. 1986.

[20] Hayat, T.; Khan, M.; Ayub, M. - *Exact Solutions of flow problems of an Oldroyd-B Fluid, Applied Mathematics and Computation,151, pp.105-119,*2004.
[21] Oldroyd, J.G. — *On the Formulation of Rheological Equations of State. Proceedings of the Royal Society of London A*, 200, pp. 523–541. 1950.
[22] Samuelson, P.A. — *Foundations of Economic Analysis.* Harvard University Press. 1947.
[23] Harrod, R.F. — *An Essay in Dynamic Theory. The Economic Journal.* March 1939.

Bibliography

Ahuja, H.L. - *Modern Economics: Micro and Macroeconomics Theory and Applications.* Sultan Chand & Sons. 2020.
Akerman, J. -*Theory of Economic Policy.* London Macmillan. 1969.
Arntz, M.; Gregory, T.; Zierahn, U. — *The Risk of Automation for Jobs in OECD Countries.* OECD. 2016.
Brynjolfsson, E.; McAfee, A. — *The Second Machine Age.* W.W. Norton & Company. 2014.
European Commission — *Digital Education Action Plan 2021–2027.* Brussels. 2021.
Harrod, R.F. — *An Essay in Dynamic Theory. The Economic Journal.* March 1939.
Hicks, J.R. - *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory.* Oxford University Press. 1946.
Keynes, J.M. — *The General Theory of Employment, Interest and Money.* Macmillan. 1936.
OECD — *The Digital Transformation of Education: Connecting Schools, Empowering Learners.* OECD Publishing. 2021.
Samuelson, P.A. - *Foundations of Economic Analysis.* Harvard University Press. 1947.
Schwab, K. — *The Fourth Industrial Revolution.* World Economic Forum. 2016.
Tapscott, D. — *The Digital Economy: Rethinking Promise and Peril in the Age of Networked Intelligence.* McGraw-Hill. 2014.
UNCTAD — *Technology and Innovation Report 2021: Catching Technological Waves –*

Innovation with Equity. United Nations. 2021.

World Bank Group — *Digital Transformation in Romania: Country Diagnostic*. 2022.

Online sources (alphabetical order):

https://ec.europa.eu/eurostat — Eurostat official website. Accessed 13-10-2025.
https://digital-strategy.ec.europa.eu — Digital Skills & Jobs Platform (European Commission). Accessed 12-10-2025.
https://www.weforum.org — World Economic Forum official website. Accessed 12-10-2025.